Math 2J - 44480

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My Information

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    - Navigate to the teaching tab and then to the Math 2J link.
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    - Place "M2J" in the title of any e-mails, I use e-mail filters.

About me

- Aerospace engineer by trade.
  - Aircraft engine design and testing.
- PhD in mathematics
  - Primary work in mathematical biology.

Course

- Infinite Series and Basic Linear Algebra
- First part - Linear Algebra
  - ~5-6 weeks
- Second part - Sequences and Series
  - ~4-5 weeks

Linear Algebra

- The study of systems of linear equations.
- Example

$$-x_1 + 2x_2 = 1$$
$$2x_1 - x_2 = 1$$

Why do we care?

- Linear systems are used to describe many things.
- Linear systems are basically the only systems we can solve!
- Nonlinear systems are very hard.

Who uses it?

- Computer Scientists
- Applied Mathematicians
- Engineers
- Statisticians

Applications

- Google search algorithm
- Statistics (ANOVA)
- Facebook
- Graphics processing - video games
- Computing engineering stresses
- Just about anything on a computer

Linear Equation

$$
\begin{aligned}
2037.6x_1 - x_2 + 12x_3 &= 4 \\
x_1 - x_2 + 77x_3 &= 0 \\
.001x_1 + x_3 &= 96
\end{aligned}
$$

$$
\begin{aligned}
2x_1 - 3x_2 &= 7 \\
-4x_1 + 6x_2 &= 11
\end{aligned}
$$

$$x_1 = 12$$

General Linear System

$$a_{1,1}x_1 + a_{1,2}x_2 + \ldots + a_{1,n-1}x_{n-1} + a_{1,n}x_n = b_1$$
$$a_{2,1}x_1 + a_{2,2}x_2 + \ldots + a_{2,n-1}x_{n-1} + a_{2,n}x_n = b_2$$
$$\vdots \qquad\qquad\qquad \vdots$$
$$a_{m,1}x_1 + a_{m,2}x_2 + \ldots + a_{m,n-1}x_{n-1} + a_{m,n}x_n = b_m$$

$a_{j,i} =$ Coefficient, Provided

$b_j =$ Constant, Provided

$x_i =$ Variable, Sought

Example

$$2x_1 - 3x_2 = 7$$
$$-4x_1 + 6x_2 = 11$$

$$a_{1,1} = 2 \qquad a_{1,2} = -3 \qquad b_1 = 7$$
$$a_{2,1} = -4 \qquad a_{2,2} = 6 \qquad b_2 = 11$$

Nonlinear System

- Everything else

$$\begin{aligned} x_1 - 2x_2^5 &= 7 \\ \sin(x_2) &= 0 \end{aligned}$$

$$\begin{aligned} x_1 - 2x_1x_2 &= 7 \\ -3x_1 + 2x_2 &= 0 \end{aligned}$$

Solution of a linear system

- A collection of numbers $x_1, x_2, \ldots, x_n$ that satisfy ALL equations

Example

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 9 \\ 2x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 - 5x_3 &= 0 \end{aligned}$$

$$\downarrow$$

$$x_1 = 1, x_2 = 2, x_3 = 3$$

- This is referred to as a 3 x 3 system

Terminology

- A system is said to be "m x n" if there are "m" equations and "n" variables / unknowns.

Terminology

- A system is said to be "consistent" if it has at least one solution.
- A system is said to be "inconsistent" if it has no solutions.

Inconsistent Example

$$x_1 + x_2 = 2$$
$$x_1 + x_2 = 3$$

- There is no x_1, x_2 that satisfies both of these.

Terminology

- Two systems are said to be “equivalent” if:
  1. They have the same number of variables.
  2. They have the same solutions.
- Equivalent systems are effectively the same system written in two different ways.

Equivalent Example

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 9 \\ 2x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 - 5x_3 &= 0 \end{aligned}$$

$$\begin{aligned} x_2 + x_2 + 2x_3 &= 9 \\ x_1 + x_2 &= 3 \\ x_3 &= 3 \end{aligned}$$

- These are consistent. They have the same number of variables the same solution

$$x_1 = 1, x_2 = 2, x_3 = 3$$